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Three dimensional Janus and time-dependent black holes

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Abstract: We show that the three dimensional Janus geometry can be embedded into the type IIB supergravity and discuss its dual CFT description. We also find exact solutions of time dependent black holes with a nontrivial dilaton field in three and higher dimensions as an application of the Janus construction.

Keywords: Black Holes, AdS-CFT Correspondence.

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Contents

1. Introduction

It is of great interest to study nonsupersymmetric deformations of $AdS_5 \times S^5$ space. The ultimate goal would be to find the gravity duals of $\mathcal{N} = 0$ Yang-Mills theory and realistic QCD models. The Janus deformation of AdS_5 space [1] is nonsupersymmetric. However, its dual field theory is not in the universality class of confining gauge theories. So in that sense it does not meet the goal that one might hope for. Nevertheless it is an interesting and rare example of nonsupersymmetric deformations where both gravitational and dual field theory descriptions are under good control. Indeed both descriptions are remarkably simple. In the gravity side the Janus deformation is a thick AdS_4-sliced domain wall in AdS_5 with the varying dilaton, where asymptotically the dilaton approaches a constant in one half of the boundary space and the different value in the other half. In the gauge theory side $\mathcal{N} = 4$ super Yang-Mills (SYM) theory is deformed by the exactly marginal operator dual to the dilaton — the SYM Lagrangian up to the total derivative — with a space dependent deformation parameter. In effect the coupling constant jumps discontinuously at the interface of two halves of the boundary space. By construction the Janus deformation preserves the $\mathrm{SO}(3, 2)$ symmetry of the AdS_4 slices. Correspondingly the dual field theory preserves the conformal symmetry at the interface, defining the interface conformal field theory (ICFT). Albeit being nonsupersymmetric, the Janus deformation was shown to be stable against a large class of perturbations and believed to be nonperturbatively stable, owing to the existence of formal killing spinors [2]. Somewhat surprisingly, an exact agreement was found even at the more quantitative level: The Janus deformation predicts the vev of the exactly marginal operator at large N and large 't Hooft coupling. Meanwhile the dual ICFT allows us to compute the vev in all orders in 't Hooft coupling by using the conformal perturbation theory. In fact two results agree at the leading order in the deformation parameter [3]. It is remarkable to find an exact agreement between two sides of the strong-weak coupling duality in the nonsupersymmetric theory.

We have been stressing the nonsupersymmetric nature of Janus and its tractability nonetheless. It is, however, worth mentioning that the supersymmetric Janus deformation was found in 5-dimensional gauged supergravity [4] and more recently in the full type IIB supergravity [5]. Correspondingly the ICFT can also be made supersymmetric by introducing the interface interactions [3]. In fact all the possible interface interactions which yield the supersymmetric ICFTs were classified in [6]. The supersymmetric Janus is interesting on its own. In particular it suggests the interpretation of the SUSY Janus in terms of the intersecting D3 and D5-branes — a potential new decoupling limit of intersecting D-branes. This may be of relevance in connection to the D-brane realization of Karch-Randall model [7].

The nonsupersymmetric Janus allows several generalizations. The Janus type domain wall exists in arbitrary dimensions and exhibits pseudo-supersymmetries [2, 8 − 10]. In the type IIB case the axion can be turned on by the $\mathrm{SL}(2, Z)$ rotation [5]. The AdS_d-sliced Janus can accommodate the $AdS_{p<d}$-sliced Janus(es) within it in a self-similar fashion [11]. By a double analytic continuation the Janus geometry can be utilized to argue a dual of the Big Bang/Crunch cosmology from the AdS/CFT perspective [12].

In this note we wish to extend our previous study of the Janus deformation to the $AdS_3 \times S^3 \times M_4$ space. The AdS_2-sliced Janus was previously discussed in [2]. Our aim is to embed it into the AdS/CFT setup in the 10d type IIB string theory. This will thus yield the Janus deformation of the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence — $\mathrm{Janus}_3/\mathrm{ICFT}_2$. It is our hope that the further simplicity due to the low dimensionality facilitates more quantitative studies and provides new qualitative perspectives.

Besides the Janus deformation of the $AdS_3 \times S^3 \times M_4$ space, we also discuss the application of the AdS_2-sliced Janus to the black hole. Exploiting the fact that the BTZ black hole is a quotient of the AdS_3 space, it is rather straightforward to consider the Janus deformation of the BTZ black hole. We will see that the Janus BTZ black hole is time dependent and has two disconnected boundaries in which the dilaton takes different constant values. We generalize this by constructing the higher dimensional black holes. The three and five dimensional solutions can be embedded into the 10d type IIB supergravity.

The paper is organized as follows. In section 2 we construct the Janus deformation of the $AdS_3 \times S^3 \times M_4$ space. In section 3 we briefly discuss its dual CFT interpretation. In section 4 we discuss the Janus deformation of the BTZ black hole. In section 5, we deal with the higher dimensional generalization of the time dependent black hole solution. We conclude our discussions in last section.

2. Janus deformation in three dimensions

In this section we would like to discuss the Janus deformation of the $\mathrm{AdS}_3 \times S^3 \times M_4$, where M_4 may be taken as either T^4 or $K3$. As we shall see below the deformation along the internal M_4 directions will be just a warping by a conformal factor related to the dilaton. Thus the details of the internal geometry do not play any role in this study. The spirit of writing down the ansatz will be pretty much the same as the case of $\mathrm{AdS}_5 \times S^5$. Along the deformation, we like to keep the $\mathrm{SO}(1,2) \times \mathrm{SO}(4)$ part out of the original $\mathrm{SO}(2,2) \times \mathrm{SO}(4)$ global symmetries. One complication is that there is a possibility of adding an extra warp factor along the internal dimensions. However, it turns out that the warp factor does not play a role of an extra degree of freedom. Rather it is determined uniquely as a function of dilaton by imposing the $\mathrm{SO}(1,2) \times \mathrm{SO}(4)$ part of the global symmetries.

We take the ansatz for the Janus solution in the Einstein frame given by

$$\begin{aligned} ds^2 &= e^{\frac{\phi}{2}} f(\mu) \left(d\mu^2 + ds^2_{AdS_2} \right) + e^{\frac{\phi}{2}}\, ds^2_{S^3} + e^{-\frac{\phi}{2}}\, ds^2_4 \,, \\ \phi &= \phi(\mu)\,, \\ F_3 &= 2 f(\mu)^{\frac{3}{2}}\, d\mu \wedge \omega_{AdS_2} + 2\omega_{S^3}\,, \end{aligned} \tag{2.1}$$

where ω_{AdS_2} and ω_{S^3} are the unit volume forms on AdS_2 and S^3 respectively. The line element ds^2_4 is for the internal manifold M_4, which may be either T^4 or $K3$.

The relevant IIB supergravity equations of motion are given by

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}\partial_\alpha \phi \partial_\beta \phi - \frac{1}{4} e^{\phi} F_\alpha{}^{\mu\nu} F_{\beta\,\mu\nu} + \frac{1}{48} e^{\phi} F^2 g_{\alpha\beta} &= 0\,, \\ \nabla^2 \phi &= \frac{1}{12} e^{\phi} F^2\,, \\ \nabla_\alpha (e^{\phi} F^{\alpha\beta\gamma}) &= 0\,, \end{aligned} \tag{2.2}$$

which should be supplemented by the Bianchi identity $dF_3 = 0$. The equation of motion for the dilaton can be integrated leading to

$$\phi'(\mu) = \frac{\gamma}{f^{\frac{1}{2}}(\mu)}\,. \tag{2.3}$$

The Einstein equations give rise to

$$\begin{aligned} f'f' - ff'' &= -2f^3 + \gamma^2 f\,, \\ f'f' - 2ff'' &= -8f^3 + 4f^2\,. \end{aligned} \tag{2.4}$$

It is easy to see that these equations are equivalent to the first order differential equation

$$f'f' = 4f^3 - 4f^2 + 2\gamma^2 f\,, \tag{2.5}$$

corresponding to the motion of a particle with zero energy in a potential given by

$$V(f) = -4f \left(f^2 - f + \frac{\gamma^2}{2} \right)\,. \tag{2.6}$$

So far we have been working with the 10d equations of motion but the above final equation may also be derived from a dimensionally reduced action down to three dimensions. We take the ansatz for the dimensional reduction as

$$\begin{aligned} ds^2 &= e^{\frac{\phi}{2}} g_{ab} dx^a dx^b + e^{\frac{\phi}{2}}\, ds^2_{S^3} + e^{-\frac{\phi}{2}}\, ds^2_4 \,, \\ \phi &= \phi(x)\,, \\ F_3 &= 2(\omega_{g_{ab}} + \omega_{S^3})\,, \end{aligned} \tag{2.7}$$

where we denote the three metric and its volume form by $ds_3^2 = g_{ab}dx^a dx^b$ and $\omega_{g_{ab}}$ respectively. The three metric and the dilaton can be a general function of the three coordinates x^a. Upon the dimensional reduction, the IIB supergravity becomes the Einstein gravity coupled to a scalar with a negative cosmological constant; the resulting action reads

$$I = \frac{1}{16\pi G_3} \int d^3x \sqrt{g_3} \left(R_3 - g_3^{ab} \partial_a \phi \partial_b \phi + 2 \right) \ . \tag{2.8}$$

where G_3 is the 3d Newton constant. We follow the convention of [13], where the 3d AdS radius and the 3d Newton constant are related to the D1/D5 charges, Q_1/Q_5, by

$$R_{\rm ads}^2 = g_6 \sqrt{Q_1 Q_5}\, l_s, \qquad G_3 = \frac{\sqrt{g_6}}{4(Q_1 Q_5)^{3/4}}\, l_s \,, \tag{2.9}$$

where the six dimensional string coupling g_6 is related to the 10d string coupling by $g_6^2 = g^2 Q_5/Q_1$. The supergravity description is valid if $g_6 Q_1$ and $g_6 Q_5$ are large but fixed. We shall set $R_{\rm ads} = 1$.

Let us solve our main equation (2.5). If $\gamma^2 > 1/2$, the geometry develops a naked curvature singularity. We shall restrict below our discussion to the case of $\gamma^2 < 1/2$ unless otherwise is mentioned specifically. The roots of the polynomial,

$$p(x) = x^2 - x + \frac{\gamma^2}{2} = (x - \alpha_+^2)(x - \alpha_-^2) \tag{2.10}$$

are given by

$$\alpha_\pm^2 = \frac{1}{2}(1 \pm \sqrt{1 - 2\gamma^2}) \tag{2.11}$$

Then the above equation can be solved by the integral

$$\mu_0 \pm \mu = \int_{\sqrt{f}}^{\infty} \frac{dx}{\sqrt{(x^2 - \alpha_+^2)(x^2 - \alpha_-^2)}} = \frac{1}{\alpha_+} \int_0^{\frac{\alpha_+}{\sqrt{f}}} \frac{dx}{\sqrt{(1 - x^2)(1 - k^2 x^2)}} \tag{2.12}$$

where $k = \alpha_-/\alpha_+$ and $\alpha_+ \mu_0 = K(k)$. We choose here μ_0 such that $\mu = 0$ at the turning point. Then the coordinate μ is ranged over the interval $[-\mu_0, \mu_0]$, where one can show that $\mu_0 \geq \pi/2$ for any γ in $[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]$. With the help of the elliptic integral of the first kind,

$$F(\varphi, k) = \int_0^{\varphi} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} \,, \tag{2.13}$$

the above integral may be represented by

$$\mu_0 \pm \mu = \frac{1}{\alpha_+} F\left(\sin^{-1}\Big(\frac{\alpha_+}{f^{\frac{1}{2}}}\Big), k \right) \ . \tag{2.14}$$

One may invert the above expression as [2]

$$f = \frac{\alpha_+^2}{\mathrm{sn}^2(\alpha_+(\mu + \mu_0), k)} \tag{2.15}$$

using the Jacobi elliptic functions, $\mathrm{sn}(z,k)$, defined by

$$z = \int_0^{\mathrm{sn}(z,k)} \frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}} \,. \tag{2.16}$$

The cosine amplitude $\mathrm{cn}(x,k)$ and the delta amplitude can be introduced by the relations,

$$\mathrm{cn}(z,k) = \cos(\sin^{-1}(\mathrm{sn}(z,k)))\,, \qquad \mathrm{dn}(z,k) = \sqrt{1-k^2\mathrm{sn}^2(z,k)}\,. \tag{2.17}$$

Then the dilaton can be integrated explicitly as

$$\phi = \phi_0 + \sqrt{2}\ln\Big(\mathrm{dn}(\alpha_+(\mu+\mu_0),k) - k\ \mathrm{cn}(\alpha_+(\mu+\mu_0),k)\Big)\,. \tag{2.18}$$

In fact using a different coordinate defined by

$$y = \int_0^{\mu} ds\sqrt{f(s)}\,, \tag{2.19}$$

the solution may be presented in terms of elementary functions. In this coordinate, the three dimensional metric g_{ab} in (2.7) takes the form

$$ds_3^2 = f(y)ds^2_{AdS_2} + dy^2\,. \tag{2.20}$$

It is straightforward to find the solution [2]

$$\begin{aligned} f(y) &= \frac{1}{2}(1+\sqrt{1-2\gamma^2}\cosh 2y)\,, \\ \phi &= \phi_0 + \frac{1}{\sqrt{2}}\ln\left(\frac{1+\sqrt{1-2\gamma^2}+\sqrt{2}\gamma\tanh y}{1+\sqrt{1-2\gamma^2}-\sqrt{2}\gamma\tanh y}\right)\,. \end{aligned} \tag{2.21}$$

Note that the boundary values of the dilaton at $\mu = \pm\mu_0$ are evaluated as

$$\phi_\pm - \phi_0 = \pm\frac{1}{\sqrt{2}}\tanh^{-1}\sqrt{2}\gamma = \pm\frac{1}{2\sqrt{2}}\ln\left(\frac{1+\sqrt{2}\gamma}{1-\sqrt{2}\gamma}\right)\,. \tag{2.22}$$

The IIB string theory has the $\mathrm{SL}(2,Z)$ duality symmetry and the classical IIB supergravity possesses the $\mathrm{SL}(2,R)$ symmetry. Hence by performing the $\mathrm{SL}(2,R)$ transformation, one may generate the new family of solutions. These solutions in general involve the nonvanishing axion χ and NS-NS three form field strength in addition. Here we shall not present the explicit form of such solutions generated by the $\mathrm{SL}(2,R)$ transformation, but would like to note that the corresponding dual CFT involves axionic domain wall together with the jump of the coupling. Namely in the dual CFT, the θ angle jumps too at the interface.

For later comparison let us compute the one-point function of dual dilaton operators. By introducing the Poincare patch parametrization for the AdS_2 part, the three metric reads

$$ds_3^2 = \frac{f(\mu)}{y^2}\left(y^2d\mu^2 - dx_0^2 + dy^2\right)\,. \tag{2.23}$$

We adopt the conformal compactification where the scaling factor is given by $\sqrt{f}/y$. Combining two halves of R^2 defined by $\mu = \pm\mu_0$, the boundary becomes a full R^2, on which the dual Janus CFT is defined. In the near boundary region, the above metric can be rewritten as

$$ds_3^2 = \frac{1}{z^2}\left(dz^2 - dx_0^2 + dx^2 + O(\frac{z^2}{x^2}dx_a dx_b)\right) , \tag{2.24}$$

where $z = y\,\mathrm{sn}(\alpha_+(\mu_0 - \mu), k)/\alpha_+$ is the inverse of the scale factor and $x = y\,\mathrm{cn}(\alpha_+(\mu_0 - \mu), k)$. Note that the dilaton behaves near the boundary as

$$\phi = \phi_\pm \mp \frac{\gamma}{2}(\mu \mp \mu_0)^2 + \cdots = \phi_\pm - \frac{\gamma}{2}\epsilon(x) z^2/x^2 + \cdots . \tag{2.25}$$

Using the AdS/CFT correspondence, we have the relation

$$\langle O_\phi \rangle = \frac{\delta I_\phi}{\delta\phi} = -\frac{1}{16\pi G_3}\sqrt{g_3}g_3^{zz}\partial_z\phi|_{z=0} = -\frac{\gamma\, Q_1 Q_5}{4\pi}\frac{\epsilon(x)}{x^2} , \tag{2.26}$$

where we take the dual operator as

$$O_\phi(z,\bar{z}) = -\frac{1}{4\pi}\sum_{i,a} :\partial X_{i,a}\bar{\partial}X_{i,a}: . \tag{2.27}$$

In ref. [14], the Fefferman-Graham coordinate system for the above metric is obtained and the boundary perturbation of the metric can be identified. Using this, one can show that

$$\langle T_{ab} \rangle = 0 , \tag{2.28}$$

which is an expected result.

3. The dual CFT

In this section we review the two dimensional CFT dual of type IIB string theory on $AdS_3 \times S^3 \times M_4$ where M_4 is either T_4 or K_3. The central charge of the CFT can be related to the AdS_3 curvature by

$$c = \frac{3R_{\mathrm{ads}}}{2G_3} . \tag{3.1}$$

This background can be obtained from a near horizon limit of Q_1 D1-branes and Q_5 D5-branes, wrapping M_4. The theory living on the D1-D5 common $1+1$ dimensional worldvolume is a $N = (4,4)$ supersymmetric field theory. The CFT dual to the near horizon limit of the D1-D5 system [15] is the IR fixed point of the $N = (4,4)$ theory. This theory can be described as a $1+1$ dimensional supersymmetric σ-model where the target space is the moduli space of Q_1 instantons in a two dimensional SU(Q_5) gauge theory. The moduli space is $4n$ dimensional where $n = Q_1Q_5$ (for $M_4 = T_4$) or $n = Q_1Q_5 + 1$ (for $M_4 = K_3$). The conformal field theory is given by [16 − 18] the smooth resolution of the orbifold CFT of the symmetric product M^n/S_n. In the following we focus on the case

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where $M_4 = T^4$. The central charge (3.1) of the CFT is then $c = 6n = 6Q_1Q_5$. The orbifold T_4^n/S_n can be constructed by starting with the free field CFT representing the tensor product T_4^n

$$S = \frac{1}{2\pi\alpha'}\int d^2z \sum_{i,a}\left(\partial X_{i,a}\bar{\partial}X_{i,a} + \psi_{i,a}\bar{\partial}\psi_{i,a} + \bar{\psi}_{i,a}\partial\bar{\psi}_{i,a}\right). \tag{3.2}$$

The indices $i = 1, 2, \cdots 4$, and $a = 1, 2, \cdots n$ parameterize n copies of the four torus T^4. Hereafter we shall set $\alpha' = 2$. The orbifold then projects onto states which are invariant under S_n acting by permutation on the coordinates $X_{i,a}$. Modular invariance mandates the inclusion of twisted sectors which contain marginal operators responsible for the smooth resolution of the orbifold singularities. The correlators of the unperturbed orbifold CFT for the bosonic fields is given by

$$\langle X^{i,a}(z,\bar{z})X^{j,b}(w,\bar{w})\rangle = -\ln \mid z-w\mid^2 \delta^{ij}\delta^{a\,b}, \tag{3.3}$$

where the sum is over permutations of the index b following the standard orbifolding procedure.

The Janus deformation of $AdS_3 \times S_3 \times M_4$ has a nontrivial profile for the six dimensional dilaton $\phi_{(6)}$. In order to identify the dual of the Janus solution, one first has to identify the operator dual to the dilaton. Symmetry considerations simplify the identification, the dilaton in the Janus solution does not depend on the coordinates of the S_3 or T_4, in the dual CFT this means that the operator transforms trivially under the SU(2) $\times$ SU(2) R-symmetry and SU(2) $\times$ SU(2) global symmetry of the $N = (4, 4)$ SCFT. Furthermore the constant Kaluza-Klein mode on the sphere of the dilaton is a massless scalar field in AdS_3 and hence corresponds to a marginal deformation with conformal dimensions $(h, \bar{h}) = (1, 1)$. A natural guess for the dual operator is therefore:

$$O_\phi(z,\bar{z}) = -\frac{1}{4\pi}\sum_{i,a} :\partial X_{i,a}\bar{\partial}X_{i,a}: \tag{3.4}$$

That the operator has the correct conformal dimensions can be seen from the two point function

$$\langle O_\phi(z,\bar{z})O_\phi(w,\bar{w})\rangle = \frac{n}{4\pi^2\mid z-w\mid^4} \tag{3.5}$$

As discussed in the previous section the solution (2.1) incorporates a Janus type AdS_2 slicing of AdS_3. The holographic dual theory is therefore an interface CFT with two half-spaces glued together by a one dimensional interface. Furthermore the dilaton takes two values $\phi_\pm$ (2.22) at the boundary $\mu = \pm\mu_0$ corresponding to the two half-spaces.

The appearance of the dilaton factor in front of the AdS_3 part of the metric (2.1) might worry the reader since this implies that asymptotically the AdS_3 curvature radius behaves as $R_{\rm ads} \sim e^{\frac{1}{2}\phi_\pm}$ near the two boundary components. Does this imply that the central charge (3.1) is jumping across the defect? This would clearly be strange since the Janus deformation is associated with a marginal operator which should not change the central charge of the CFT. The resolution of this puzzle lies in the fact that the three dimensional

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Newton's constant is behaving like $G_3 \sim e^{\frac{1}{2}\phi_\pm}$ near the boundary and the dilaton factors cancel out in the formula for the central charge.

The AdS_3 Janus deformation can be analyzed using conformal perturbation theory. For the location of the interface at $x_2 = 0$, where $z = x_1 + ix_2$, the deformation is defined by adding the following term to the action

$$S = S + \lambda \int d^2 z \epsilon(x_2) O_\phi(z, \bar{z}) \tag{3.6}$$

where $\lambda = \gamma + O(\gamma^2)$. We can apply conformal perturbation theory method which was applied for the AdS_5 Janus solution in [3]. We will only calculate the simplest correlation functions which provide nontrivial checks of the correspondence[1]. First, it is clear that the expectation value $\langle O_\phi(z, \bar{z})\rangle = 0$ of the operator (3.4) vanishes in the unperturbed theory since the operator O_ϕ is normal ordered. Second the one point function of (3.4) to order $o(\lambda)$ is given by

$$\begin{aligned} \langle O_\phi(w, \bar{w})\rangle_\lambda &= \lambda \langle O_\phi(w, \bar{w}) \int d^2 z \epsilon(x_2) O_\phi(z, \bar{z})\rangle + o(\lambda^2) \\ &= -\frac{\gamma n}{4\pi} \frac{\epsilon(w_2)}{\mid w \mid^2} \end{aligned} \tag{3.7}$$

Third the expectation value of the energy momentum tensor to first order in λ is given by

$$\begin{aligned} \langle T(w)\rangle_\lambda &= \lambda \langle T(w) \int d^2 z \, \epsilon(x_2) O_\phi(z, \bar{z})\rangle = 0 \\ \langle \bar{T}(\bar{w})\rangle_\lambda &= \lambda \langle \bar{T}(\bar{w}) \int d^2 z \, \epsilon(x_2) O_\phi(z, \bar{z})\rangle = 0 \end{aligned} \tag{3.8}$$

4. Time dependent BTZ-type black hole

In this section, we would like to present another type of related gravity solution. This solution describes a black hole of a BTZ type. But the black hole solution has an unconventional character. The horizon size and the dilaton value on the horizon are time dependent. The geometry involves two disconnected boundaries and the couplings of the boundaries differ from each other. A similar kind of multi boundary solution in the Euclidean context is mentioned in ref. [20] but the geometry there is not directly related the one presented here.

The construction of the solution goes as follows. Note that the ansatz in (2.1) may be equivalently presented as

$$ds_3^2 = f(\mu) \cos^2 \mu \, ds^2_{AdS_3} \, , \quad \phi = \phi(\mu) \, , \tag{4.1}$$

with the AdS_3 metric

$$ds^2_{AdS_3} = \frac{1}{\cos^2 \mu}(d\mu^2 + ds^2_{AdS_2}) \, . \tag{4.2}$$

[1]In ref. [19], some of the correlation functions are computed exactly for the ICFT.

Then, of course, the dilaton and the conformal factor are solved by the Janus solution of the previous section. As in the AdS_5 case, the AdS_3 becomes the global/Poincare metric if one uses the global/Poincare parametrization for the AdS_2. In fact one may replace $ds^2_{AdS_3}$ by any three metric satisfying $R_{ab} = -2g_{ab}$, where a translation in μ should be isometry of the metric $\cos^2\mu\, g_{ab}$. The AdS_3 metric for instance can be replaced by the metric for any BTZ black hole. Here we illustrate the detailed construction for the zero angular momentum case only. The BTZ black hole solution may be constructed using the orbifolding technique. Note that AdS_3 space is the hyperboloid in $R^{2,2}$ satisfying

$$-Y_0^2 - Y_3^2 + Y_1^2 + Y_2^2 = -1\,. \tag{4.3}$$

We then use the parametrization of the AdS_3 space by

$$\begin{aligned}
Y_0 &= \sqrt{\frac{r^2}{r_0^2}-1}\,\sinh r_0 t\,, \qquad & Y_2 &= \pm\sqrt{(r/r_0)^2-1}\,\cosh r_0 t \\
Y_1 &= \frac{r}{r_0}\sinh r_0\theta\,, & Y_3 &= \pm\frac{r}{r_0}\cosh r_0\theta\,.
\end{aligned} \tag{4.4}$$

The metric takes the form of

$$ds^2_{\rm BTZ} = -(r^2-r_0^2)dt^2 + \frac{dr^2}{r^2-r_0^2} + r^2 d\theta^2\,. \tag{4.5}$$

With the identification $\theta \sim \theta + 2\pi$, the above describes the BTZ black hole with vanishing angular momentum [21]. The horizon is at $r = r_0$ and $r = 0$ corresponds to a singularity of the orbifold type. In the solution, the coordinate μ is related to the BTZ coordinates by

$$\tan\mu = Y_2 = \pm\sqrt{(r/r_0)^2-1}\,\cosh r_0 t\ . \tag{4.6}$$

The BTZ coordinate does not cover the whole region of our geometry; it can be also extended to the asymptotic region in addition to the region beyond the horizon. To see this, let us introduce the Kruskal coordinates

$$V = e^{r_0(t+r_*)}\,, \qquad U = -e^{-r_0(t-r_*)}\,, \tag{4.7}$$

where r_* denotes

$$r_* = \frac{1}{2r_0}\ln\left(\frac{r-r_0}{r+r_0}\right)\,. \tag{4.8}$$

The coordinates r and μ are related to (U,V) by

$$\frac{r}{r_0} = \frac{1-UV}{1+UV}\,, \qquad \cos^2\mu = \frac{(1+UV)^2}{(1+U^2)(1+V^2)}\,. \tag{4.9}$$

In this coordinate, the three metric becomes

$$ds_3^2 = \frac{f(\mu)}{(1+U^2)(1+V^2)}(-4dUdV + r_0^2(1-UV)^2 d\theta^2)\,. \tag{4.10}$$

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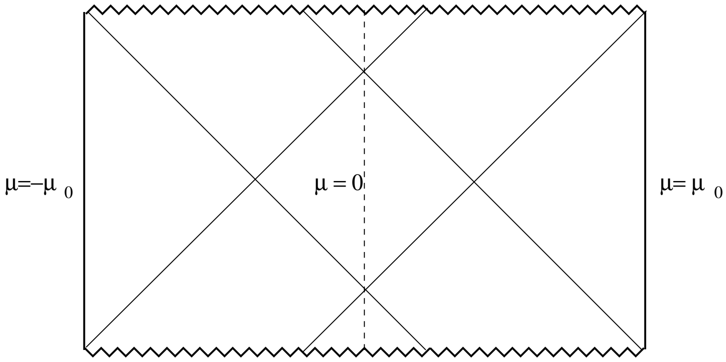

Figure 1: Penrose diagram for the time dependent black hole with two couplings. The τ ($\in [-\pi/2, \pi/2]$) coordinate runs vertically upward and μ ($\in [-\mu_0, \mu_0]$) to the right horizontally

with U, $V \in (-\infty, \infty)$ as a result of the extension. But even this new coordinate does not cover the whole geometry and can be extended further to the asymptotic region. The fully extended geometry may be obtained by introducing a parametrization,

$$V = \tan w_1 \,, \qquad U = \tan w_2 \;. \tag{4.11}$$

The metric now takes the form

$$\begin{aligned} ds_3^2 &= f(\mu)(-4dw_1 dw_2 + r_0^2 \cos^2(w_1 + w_2) d\theta^2) \\ &= f(\mu)(-d\tau^2 + d\mu^2 + r_0^2 \cos^2 \tau d\theta^2) \,, \end{aligned} \tag{4.12}$$

where $\tau = w_1 + w_2$ and $\mu = w_1 - w_2$. One may use $y = \int_0^\mu ds \sqrt{f(s)}$ that is introduced before and the metric is then represented in terms of elementary functions by

$$ds_3^2 = dy^2 + \frac{1}{2}\left(1 + \sqrt{1 - 2\gamma^2} \cosh 2y\right)(-d\tau^2 + r_0^2 \cos^2 \tau \, d\theta^2) \,, \tag{4.13}$$

with the scalar field given in (2.21). The orbifold singularity is now at $\tau = \pm\pi/2$ and the asymptotic spatial infinities are located at $\mu = \pm\mu_0$. Thus the Penrose diagram is covering the region $\tau \in [-\pi/2, \pi/2]$ and $\mu \in [-\mu_0, \mu_0]$.

The coupling on the right/left side boundary takes the value of e^{ϕ_+} / e^{ϕ_-}. Two different boundary CFT's are correlated through the bulk in a subtle manner [20]. The boundary CFT is not the Janus type. Rather the coupling is uniform on the entire circle and remains constant in time. The geometry has the time reversal symmetry at $\tau = 0$ axis and a parity symmetry under the interchange of μ to $-\mu$. In the right asymptotic regions with $\tau \geq 0$, the future-horizon area is described by the points $\mu = \mu_0 - \pi/2 + \tau$ with $\tau \in [0, \pi/2]$. The horizon area may be evaluated as

$$A(\tau) = 2\pi r_0 \frac{\alpha_+ \sin(\pi/2 - \tau)}{\mathrm{sn}(\alpha_+(\pi/2 - \tau), k)} \,. \tag{4.14}$$

For $\tau \in [0, \pi/2]$, the area grows monotonically in time and the minimum value is $2\pi r_0 \frac{\alpha_+}{\mathrm{sn}(\frac{\pi\alpha_+}{2}, k)}$ and the maximum, $2\pi r_0$.

Finally the case of $\gamma^2 \geq 1/2$ shall be treated when we consider the black holes in general dimensions.

5. Time dependent topological black holes in higher dimensions

In this section, we would like to generalize the black hole solution of previous section to higher dimensions. The construction is again fairly straightforward. We begin with a dilaton Einstein gravity described by

$$I = \frac{1}{16\pi G_d} \int \sqrt{g_d} \left(R_d - g_d^{ab} \partial_a \phi \partial_b \phi + (d-1)(d-2) \right) . \tag{5.1}$$

with $d \geq 3$. As we just described in the previous sections, any solutions of the above action for the $d = 3$ case can be embedded into the 10d type IIB supergravity. This is also true for the $d = 5$ case, which leads to a deformation of $AdS_5 \times S^5$ geometry. The ansatz may be taken as

$$ds_d^2 = f(\mu)(d\mu^2 + ds_{d-1}^2) , \quad \phi = \phi(\mu) , \tag{5.2}$$

where the $(d-1)$ dimensional metric $\tilde{g}_{ij}$ describes an Einstein space satisfying $\tilde{R}_{ij} = -(d-2)\tilde{g}_{ij}$. The equation of motion for the dilaton can be integrated leading to

$$\phi'(\mu) = \frac{\gamma}{f^{\frac{d-2}{2}}(\mu)} , \tag{5.3}$$

and the Einstein equations are reduced to

$$f'f' = 4f^3 - 4f^2 + \frac{4\gamma^2}{(d-1)(d-2)} f^{4-d} . \tag{5.4}$$

This can be solved by the integral

$$\mu_0 \pm \mu = \int_f^\infty \frac{dx}{2\sqrt{x^3 - x^2 + \frac{\gamma^2}{(d-1)(d-2)} x^{4-d}}} , \tag{5.5}$$

where μ_0 is chosen such that $\mu = 0$ at the turning point. Here we are discussing the case of $\gamma^2 \leq \gamma_c^2$ with $\gamma_c^2 = (d-2)\left(\frac{d-2}{d-1}\right)^{d-2}$, for which the geometry is free of timelike curvature singularity. Then the coordinate μ is ranged over the interval $[-\mu_0, \mu_0]$ with $\mu_0 \geq \pi/2$.

Up to this point, there is no difference from the construction of the Janus solutions except that we put the $(d-1)$ dimensional spacetime in a generic form of the Einstein manifold with negative cosmological constant. Now the trick is to take $\tilde{g}_{ij}$ as

$$ds_{d-1}^2 = -d\tau^2 + \cos^2\tau ds_\Sigma^2 \tag{5.6}$$

where ds_Σ^2 is describing the compact, smooth, finite volume Einstein space metric in $(d-2)$ dimensions satisfying $R^\Sigma_{kl} = -(d-3)g^\Sigma_{kl}$. One example of such space is given by the quotient of the hyperbolic space H_{d-2} by a discrete subgroup of the hyperbolic symmetry group, SO$(1, d-2)$. One can pick the subgroup Γ such that $\Sigma_{d-2} = H_{d-2}/\Gamma$ is a compact, smooth, finite volume space. Notice that Σ_2 constructed this way corresponds to constant curvature Riemann surface of genus no less than two.

The resulting metric,

$$ds_d^2 = f(\mu)(-d\tau^2 + d\mu^2 + \cos^2\tau ds_\Sigma^2)\,, \tag{5.7}$$

is the d dimensional generalization of the three metric in (4.12). Note here that $\tau \in [-\pi/2, \pi/2]$ as before. Therefore the Penrose digram for this higher dimensional black hole is again described by figure 1, in which a point represents Σ slice. The spacetime is locally isomorphic to AdS_d and the curvature singularity at $\tau = 0$ is again of the orbifold type.

Let us turn to the over-critical case of $\gamma^2 > \gamma_c^2$. The scale factor $f(\mu)$ is now ranged over $[0, \infty)$ without any turning point and the geometry involves an extra curvature singularity at $f = 0$, which is timelike. The factor $f(\mu)$ can be solved by the integral

$$\mu_0 - \mu = \int_f^\infty \frac{dx}{2\sqrt{x^3 - x^2 + \frac{\gamma^2}{(d-1)(d-2)}x^{4-d}}}\,. \tag{5.8}$$

where μ_0 can be taken to be arbitrary. Since f has to be non negative, the μ coordinate is ranged over $[\mu_0 - \kappa, \mu_0]$ where the length of the interval, $\kappa(\gamma^2)$, is determined by the integral

$$\kappa(\gamma^2) = \int_0^\infty \frac{dx}{2\sqrt{x^3 - x^2 + \frac{\gamma^2}{(d-1)(d-2)}x^{4-d}}}\,. \tag{5.9}$$

For $\gamma^2 > \gamma_c^2$, $\kappa(\gamma^2)$ is decreasing monotonically from infinity to zero. The metric for the geometry is still given by (5.7) but the timelike singularity occurs at $\mu = \mu_0 - \kappa$. Because of this, the spacetime cannot be extended to the region of $\mu < \mu_0 - \kappa$. If $\gamma_c^2 < \gamma^2 < \gamma_s^2$ with γ_s^2 defined by $\pi/2 = \kappa(\gamma_s^2)$, the geometry becomes free of naked singularity describing a regular time-dependent black hole.

Representing the volume of Σ space by $\mathcal{V}_\Sigma$, the future-horizon area is given by

$$A(\tau) = \mathcal{V}_\Sigma(\cos(\tau) f^{\frac{1}{2}}(\mu_0 + \tau - \pi/2))^{d-2}\,. \tag{5.10}$$

One can check that the area is monotonically increasing for $\tau \in [-\pi/2, \pi/2]$ reaching the maximal value $\mathcal{V}_\Sigma$ at $\tau = \pi/2$. In this sense, the black hole is truly time dependent for nonvanishing γ. For $\gamma = 0$, the horizon area remains constant and the black hole becomes static. In fact, it is straightforward to show that the $\gamma = 0$ solution corresponds to the $M = 0$ and $k = -1$ topological black hole solution of ref. [22].

We shall not discuss the detailed framework for the gravity/gauge theory correspondence here. Note, however, that the boundary metric of the dual CFT can be obtained by the multiplication of any h^2 where h approaches linearly zero at the boundary. Different choice of h leads to a different boundary metric. By taking $h = f^{-\frac{1}{2}}$ for instance, the boundary metric becomes

$$ds_B^2 = -d\tau^2 + \cos^2\tau ds_\Sigma^2\,, \tag{5.11}$$

which is cosmological. The dual CFT will be defined in this cosmological background spacetime. Although the boundary spacetime reveals the big-bang and big-crunch singularities at $\tau = \pm\pi/2$, the bulk-extended metric at the points is perfectly regular.

If one chooses $h = f^{-\frac{1}{2}}/\cos\tau$, the boundary now becomes

$$ds_B^2 = -dt^2 + ds_\Sigma^2 \,, \tag{5.12}$$

where $t \in (-\infty, \infty)$. Thus the dual CFT is defined on $R \times \Sigma$ now.[2] In this case, the finite temperature system starts off with some out of equilibrium state at $\tau = 0$ and then the excess kinetic energy is thermalized reaching the equilibrium at late time. One expects that detailed information about the thermalization process can be extracted from the behavior of the solution. Further study is necessary in this direction.

6. Conclusions

We generalized our previous study of the Janus deformation to the $AdS_3 \times S^3 \times M_4$ space. The AdS_3 part is replaced by the AdS_2-sliced Janus. However, the total spacetime is not the simple product of $\text{Janus}_3 \times S^3 \times M_4$, as one might have thought. Indeed each component of the product space is warped in a specific manner by an exponential of the dilaton. Besides this nontriviality, the solution is expressed in a simple analytic form. Thus one may hope that further quantitative studies will be much facilitated. The dual CFT interpretation is similar to the $AdS_5 \times S^5$ case. A spatially dependent marginal deformation dual to the dilaton leaves the conformal invariance only in the interface of two halves of the boundary space. The resulting dual field theory is an ICFT_2.

Apart from the Janus deformation of the $AdS_3 \times S^3 \times M_4$ space, we also discussed the Janus deformation of the BTZ black hole. The Janus BTZ black hole turned out to be time dependent and has two disconnected boundaries. The dilaton does not divide each boundary component into two halves. Rather, it takes one value in one component of the boundaries and the other in the other component. This black hole solutions can be generalized to the higher dimensions. Among these the three and five dimensional ones can be embedded into the type IIB supergravity. It would be quite interesting to study further the microscopic description of these time dependent black holes.

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